

Sirindhorn International Institute of Technology
Thammasat University at Rangsit
School of Information, Computer and Communication Technology

TCS 455: Problem Set 2

Semester/Year: 2/2009

Course Title: Mobile Communications

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Course Web Site: <http://www.siit.tu.ac.th/prapun/ecs455/>

Due date: Nov 27, 2009 (Friday)

Instructions

1. ONE question will be graded. Of course, you do not know which problem will be selected; so you should work on all of them.
2. Late submission will not be accepted.
3. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Please submit your solutions for the following questions. You may use the MATLAB code from

<http://infohost.nmt.edu/~borchers/erlang.html>

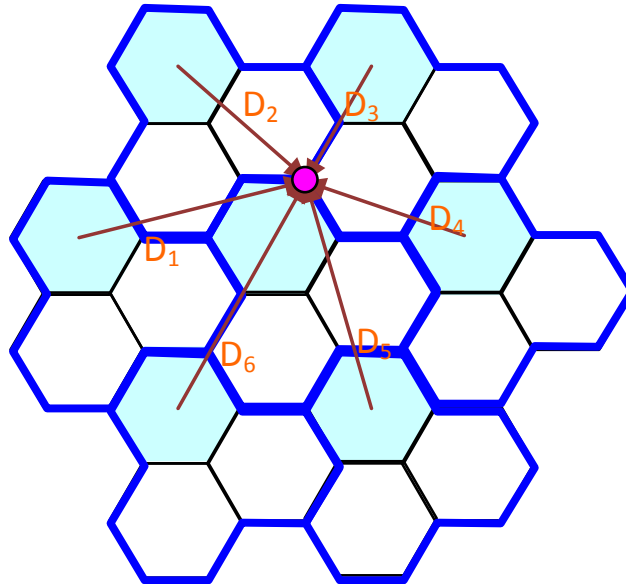
to evaluate the Erlang B formula. Figure 3.6 from the textbook, which is included here in Appendix A, may give a rough approximation.

1. Some possible values of cluster size is $N = 3, 4, \text{ or } 7$. Find the next fifteen lowest values of N .
2. If 20 MHz of total spectrum is allocated for a **duplex** wireless cellular system and each **simplex** channel has 25 kHz RF bandwidth, find:
 - (a) the number of duplex channels.
 - (b) the total number of channels per cell site, if $N = 4$ cell reuse is used.
3. In this question, we will find the unsectorized (using omnidirectional antennas) SIR value when $N = 3$.

(a) Recall that the SIR can be calculated from

$$\frac{S}{I} = \frac{kR^{-\gamma}}{\sum_{i=1}^K kD_i^{-\gamma}}$$

where D_i is the distance from the i th interfering co-channel cell base station. Find all the distance D_i in the figure below.



Express them as a function of R (the distance from the center of the hexagon to its vertex.)

Hint: $D_1 = \sqrt{13}R$.

(b) Calculate the SIR (in dB) using the D_i values that you got from part (a). Assume a path loss exponent of $\gamma = 4$.

(c) Approximate all value of D_i by the **center-to-center** distance D between the nearest co-channel. Express D as a function of R . Recalculate the SIR (in dB) with all D_i replaced by D .

(d) Compare your answers from part (b) and part (c).

4. A cellular service provider decides to use a digital TDMA scheme which can tolerate a signal-to-interference ratio of 15 dB in the worst case. Find the optimal value of N for (a) omnidirectional antennas, (b) 120° sectoring, and (c) 60° sectoring. (Assume a path loss exponent of $\gamma = 4$.)

Hint: Approximate the SIR value by

$$\frac{S}{I} = \frac{kR^{-\gamma}}{K \times (kD^{-\gamma})} = \frac{1}{K} \left(\frac{D}{R} \right)^\gamma = \frac{1}{K} (\sqrt{3N})^\gamma$$

where K is the number of (first-tier) interfering (co-channel) base stations.

5. How many users can be supported for 0.5% blocking probability for the following number of trunked channels in a blocked calls cleared system?
- (a) 5
 - (b) 15
 - (c) 25

Assume each user generates 0.1 Erlangs of traffic.

6. Assume each user of a single base station mobile radio system averages three calls per hour, each call lasting an average of 5 minutes.
- (a) What is the traffic intensity for each user?
 - (b) Find the number of users that could use the system with 1% blocking if only one channel is available.
 - (c) Find the number of users that could use the system with 1% blocking if five trunked channels are available.
 - (d) If the number of users you found in (c) is suddenly doubled, what is the new blocking probability of the five channel trunked mobile radio system?

7. This question begins with some background concepts in part (a). Read it carefully and then work on the remaining parts.

- a. For communications theory, there are two important types of random variables that are derived from Gaussian random variables.

The first type of random variable is a Rayleigh random variable. Let x_1 and x_2 be independent Gaussian random variables each with zero mean and variance σ^2 . Let y be defined by $y = \sqrt{x_1^2 + x_2^2}$. By definition, y is then a Rayleigh random variable.

(Notice that I could also define y as $y = |x_1 + jx_2|$ so y is definitely connected with complex numbers). After some computation, one finds that the probability density function for a Rayleigh random variable, denoted by $f_{\text{Rayleigh}}(y)$, is

$$f_{\text{Rayleigh}}(y) = u(y) \frac{y}{\sigma^2} \exp\left(-\frac{y^2}{2\sigma^2}\right)$$

where $u(y)$ is the unit step function.

The second type of random variable is a Rician random variable. Let x_1 and x_2 be independent Gaussian random variables where x_1 has mean m_1 and variance σ^2 and x_2 has mean m_2 and variance σ^2 . Notice that the two variances are the same. Let y be defined by $y = \sqrt{x_1^2 + x_2^2}$. By definition, y is then a Rician random variable.

(Notice that I could also define y as $y = |x_1 + jx_2|$ so y is definitely connected with

complex numbers). After some computation, one finds that the probability density function for a Rician random variable, denoted by $f_{\text{Rician}}(y)$, is

$$f_{\text{Rician}}(y) = u(y) \frac{y}{\sigma^2} \exp\left(-\frac{1}{2\sigma^2}(y^2 + A_m^2)\right) I_0\left(\frac{yA_m}{\sigma^2}\right)$$

where $u(y)$ is the unit step function, $I_0(z)$ is the modified Bessel function of the first type of order 0, and A_m is defined by $A_m = \sqrt{m_1^2 + m_2^2}$ or equivalently by $A_m = |m_1 + jm_2|$.

Probabilities are basically the frequencies of events and expectations are basically averages. So, you can estimate the probability density function for a Rayleigh random variable by doing the following calculations:

- (i) Compute large numbers of independent Gaussian random numbers and associate them in pairs.
- (ii) From each pair compute a Rayleigh random number (by taking the square root of the sum of the squares of the two numbers in the pair).
- (iii) Make a histogram of the Rayleigh random numbers.

The only thing you must be careful about is the scaling of the histogram: remember that

$$P[\alpha \leq y \leq \alpha + \delta] \approx f_{\text{Rayleigh}}(y) \times \delta$$

so you need to scale the histogram by the bin width δ in addition to scaling it by the number of pairs of random numbers that you use. (You can always check such scalings because the integral from $-\infty$ to $+\infty$ of the probability density function must equal 1).

One can compute an estimate of the expectation of some arbitrary function g of a Rayleigh random variable by doing very similar calculations. These types of estimates are called "sample expectations". The calculations are:

- (i) Compute large numbers of independent Gaussian random numbers and associate them in pairs.
- (ii) From each pair compute a Rayleigh random number (by taking the square root of the sum of the squares of the two numbers in the pair).
- (iii) Compute the average of g applied to the Rayleigh random numbers you computed in the previous part. You must be careful to normalize the average by the number of terms in the sum.

b. Write a MATLAB program that will compute three things for an arbitrary Rayleigh random variable:

- (i) The histogram that approximates the probability density function.

(ii) The exact probability density function $f_{\text{Rayleigh}}(y) = u(y) \frac{y}{\sigma^2} \exp\left(-\frac{y^2}{2\sigma^2}\right)$.

(iii) The sample mean, second moment, and variance.

By “arbitrary Rayleigh random variable” I mean a program that works for an arbitrary value of σ^2 .

You are in luck, because I've written the program for $\sigma^2 = 1$! The MATLAB script is

```
nrealizations=1000;
nbins=50;
x=randn(nrealizations,1) + j.*randn(nrealizations,1);
y=abs(x);
%deltabins=(max(y)-min(y))/nbins;
deltabins=max(y)/nbins;
%bins=deltabins*[0:1:nbins]+min(y);
bins=deltabins*[0:1:nbins];
counts=histc(y,bins);
counts=counts./(nrealizations .* deltabins);
p=rayleigh(bins,1);
hold on
bar(bins,counts,'histc')
plot(bins,p)
hold off
xlabel('y');
ylabel('Rayleigh p(y) for \sigma^2=1');
Ey=sum(y)./nrealizations;
Eyy=sum(y.*y)./nrealizations;
Vary=Eyy-Ey.^2;
fprintf(1, ...
'For y a Rayleigh random variable with sigma^2=1 and using %d\n', ...
nrealizations);
fprintf(1, ...
'realizations to compute the sample statistics, we find that the\n');
fprintf(1, ...
'sample mean, second moment, and variance are %10.6f %10.6f %10.6f\n', ...
Ey,Eyy,Vary);
fprintf(1, ...
'respectively.\n');
```

which calls another MATLAB function stored in a file named `rayleigh.m` which is

```
function p=rayleigh(x,sigmasquared)
%function p=rayleigh(x,sigmasquared)
p = (x>=0.0) .* x .* exp(-0.5 .* x.^2 ./ sigmasquared) ./ sigmasquared;
```

So all you have to do is modify the script to incorporate arbitrary values for σ^2 . Hint: remember that if v has probability density function $\mathcal{N}(0,1)$ then $\sigma v + m$ has probability density function $\mathcal{N}(m, \sigma^2)$.

Print out all of your MATLAB codes and highlight the differences between your codes and my codes above. Also, turn in your plots and sample expectations (sample mean, second moment, and variance) for the case when $\sigma^2 = 20$.

- c. Write a MATLAB program that does the same thing (as in part (b) but) for a Rician random variable. It will be necessary to use the MATLAB function `besseli` in order to compute the Bessel function. Type “`help besseli`” to get more information. Or you can examine my MATLAB function stored in a file named `rician.m`:

```
function p=rician(x,sigmasquared,Am)
%function p=rician(x,sigmasquared,Am)
p = (x>=0.0) .* x .* exp( -0.5 .* (x.^2 + Am.^2) ./ sigmasquared ) ...
.* besseli(0,x .* Am ./ sigmasquared) ./ sigmasquared;
```

Print out all of your MATLAB codes and highlight the differences between your codes and my codes above.

Also, turn in your plots and sample expectations (sample mean, second moment, and variance) for the Rician case for two sets of parameters which are ($m_1 = 3, m_2 = 4, \sigma^2 = 20$) and ($m_1 = 3, m_2 = 4, \sigma^2 = 1$).

Appendix A

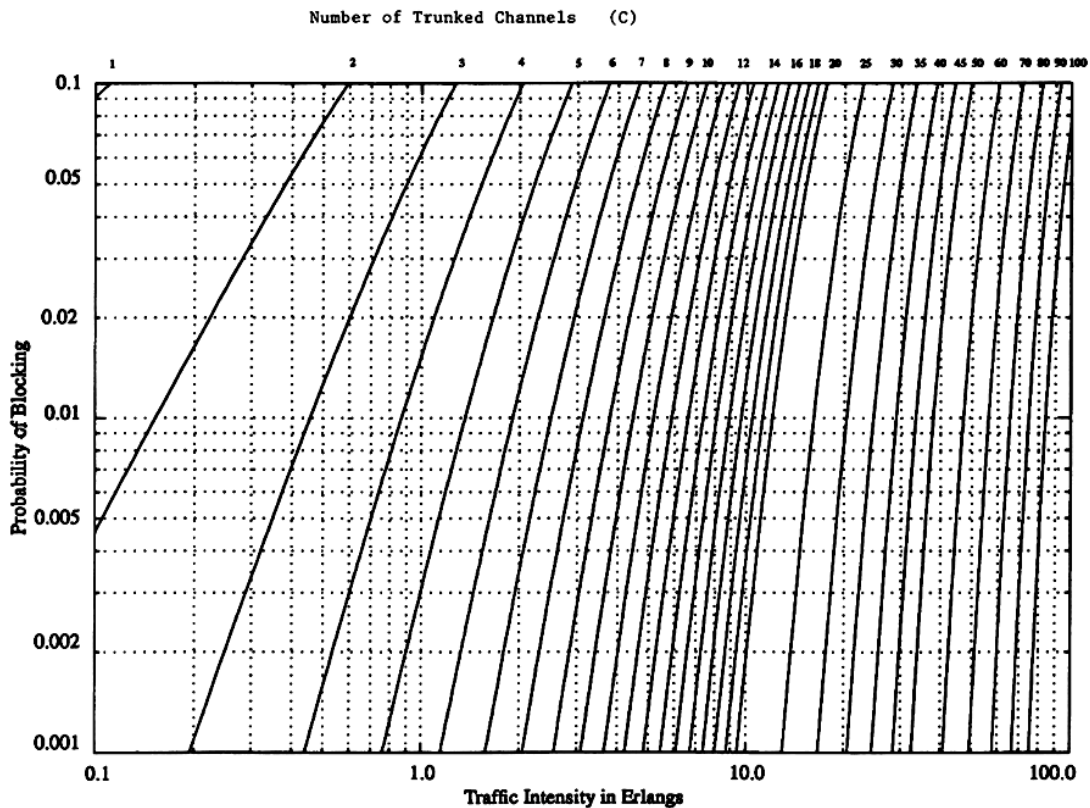


Figure 3.6 The Erlang B chart showing the probability of blocking as functions of the number of channels and traffic intensity in Erlangs.